

Final exam sheet for 'Atmospheric Dynamics'.

Questions

1. (10 points)

The thermal wind equation in the pressure coordinate system is

$$\frac{\partial \mathbf{v}}{\partial \ln p} = -\frac{R}{f} \mathbf{k} \times \nabla T \quad (1)$$

- a) Which two basic approximate balances valid for large-scale motion lead to the thermal wind equation 1? Write the equations for these balances and derive equation 1.
- b) Bob lives at a latitude of 45 degrees north. He wakes up one morning and measures a temperature of 10 C and no wind. His friend Bill lives 1000 km east and measures a temperature of 0 C and no wind as well. When Bob and Bill look up to the sky they see clouds. Assume that the horizontal temperature changes are constant with height. Are the clouds moving south or north? Assuming that the clouds are at 500 hPa and the surface pressure is 1000 hPa how fast are they moving? Show all your work and state your assumptions carefully. (The gas constant for air is $R = 287 \text{ J K}^{-1} \text{ kg}^{-1}$).
- c) Calculate the temperature advection by the thermal wind at 500 hPa $\mathbf{v}_T \cdot \nabla T$ that you calculated in b), assuming ∇T to be constant with height.

2. (10 Points)

Consider the conservation of barotropic potential vorticity

$$\frac{d_h(\xi + f)}{dt} = 0 \quad . \quad (2)$$

- a) State the conditions under which Eq. (2) becomes the barotropic vorticity equation

$$\frac{d_h(\xi + f)}{dt} = 0 \quad . \quad (3)$$

- b) Using the approximations $\xi = \nabla^2\Phi/f_0$ and $\mathbf{v} = f_0^{-1}\mathbf{k} \times \nabla\Phi$ show that the barotropic vorticity equation (3) can be written in the form

$$\frac{d_h}{dt}\nabla^2\psi + \beta\frac{\partial\psi}{\partial x} = 0 \quad . \quad (4)$$

- c) Linearize Eq. (4) around a basic state that fulfills $\bar{\psi} = -\bar{u}y + \text{const.}$, where \bar{u} is constant and assuming only zonally and time dependent streamfunction perturbations $\psi'(x, t)$. Show that the linearized barotropic vorticity equation is given by

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\frac{\partial^2\psi'}{\partial x^2} + \beta\frac{\partial\psi'}{\partial x} = 0 \quad , \quad (5)$$

where β is assumed to be constant.

- d) Derive the phase speed of zonally propagating Rossby waves (assume solutions of type: $\psi' = Ae^{i(kx - \nu t)}$). What is the phase speed if $\bar{u} = 10$ m/s, $\beta = 1 \cdot 10^{-11}\text{s}^{-1}\text{m}^{-1}$, and the wavelength is $L_x = 3.14 \cdot 10^6$ m? (assume $\pi \approx 3.14$)
- e) Calculate the wavelength of stationary Rossby waves. Which wavelength results for $\beta = 1 \cdot 10^{-11}\text{s}^{-1}\text{m}^{-1}$, and $\bar{u} = 10$ m/s (assume $\pi \approx 3.14$)?
- f) Calculate the group velocity of purely zonally propagating Rossby waves and the group velocity for the special case of stationary Rossby waves.
- g) What modifications in the vorticity equation 4 and in the basic state introduced in c) are needed to support baroclinic instability? What other equation is needed to close the system in that case?
- h) A stationary Rossby wave generated by an equatorial heating is moving northeastward. Assuming the zonal stationary wavenumber to be constant, explain why there is a latitude where the stationary meridional wavenumber becomes 0. What is the name of this latitude?

3. (6 points)

- a) For a motionless stationary state along the equator, the following equation for the thermocline is valid:

$$\frac{\partial h}{\partial x} = \frac{1}{\rho h g'}\tau_x \quad . \quad (6)$$

Explain the meaning of all variables and constants in this equation.

b) Assume a mean wind stress distribution along the equator:

$$\begin{aligned}\tau_x &= 0 \quad \text{for } \text{lon} \leq 180 \text{ E} \\ \tau_x &= -0.05 \text{ N/m}^2 \quad \text{for } 180 \text{ E} \leq \text{lon} \leq 240 \text{ E} \\ \tau_x &= 0 \quad \text{for } \text{lon} \geq 240 \text{ E}\end{aligned}$$

Using the approximation 6, calculate the thermocline distribution along the equator. Assuming that the thermocline depth at the western edge is 100 m, what is the approximate total change in height between 180 E and 240 E? (Assume the radius of the Earth is $r = 6.37 \cdot 10^6$ m).

c) In the eastern equatorial Pacific a warm Sea Surface temperature anomaly is observed that develops into a El Nino event. Explain the atmospheric adjustment processes that follow and eventually lead to the positive atmospheric feedback mechanism. Explain also the positive ocean feedback mechanism that follows. What is the name of this positive atmosphere-ocean coupled feedback mechanism?

4. (4 points)

The stationary equation for the vertical component of the zonal mean flow is

$$-S_p \bar{\omega} = -\frac{\partial \overline{v'T'}}{\partial y} + \frac{\bar{Q}}{c_p} \quad (7)$$

- Explain the meaning of each term in this equation.
- What are the main terms responsible for the Hadley Circulation. Draw the Hadley circulation in a latitude-vertical section. Indicate where rising and sinking motion can be found.

5. (4 points)

Consider the Lorenz Equations

$$\begin{aligned}\dot{X} &= p(Y - X) \\ \dot{Y} &= -XZ + rX - Y \\ \dot{Z} &= XY - bZ .\end{aligned}\tag{8}$$

Show that the stationary state at rest $X = Y = Z = 0$ becomes unstable if the 'Rayleigh number', r , is larger than one. (Hint: Linearize the system 8 around the stationary state at rest, $X = Y = Z = 0$, then insert fourier components for the perturbation quantities $X' = X_0 e^{mt}$, $Y' = Y_0 e^{mt}$, $Z' = Z_0 e^{mt}$, and calculate the characteristic equation).

6. (6 points)

The atmosphere has a spectrum of modes of variability covering a large range of periods, ranging from days to decades.

- a) Hasselmann has proposed a model for the interaction of the atmosphere with an upper ocean layer:

$$\rho c H \frac{dT}{dt} = N - \lambda T \quad ,\tag{9}$$

Explain all terms in this equation.

- b) Assuming the noise to be $N = N_\nu e^{i\nu t}$, show that the power spectrum can be expressed as:

$$|T_\nu|^2 = \frac{N_\nu^2}{\lambda^2 + (\nu \rho c H)^2} \quad .\tag{10}$$

Use this equation to explain a reddening of the spectrum if the atmospheric modes interact with an Ocean layer.

- c) Assume $\rho = 1000 \text{ kg/m}^3$, $c = 4186 \text{ J/kg}$, $H = 50 \text{ m}$, $\lambda = 15 \text{ W/(m}^2 \text{K)}$, $N_\nu = 50 \text{ W/m}^2$ Calculate the resulting amplitude $|T_\nu|$ for 2 atmospheric noise modes, one with 720 day and the other with 20 day period.