6 The general circulation

The *general circulation* of the atmosphere is usually considered to include the totality of motions that characterise the global-scale atmospheric flow. Climate dynamics is one if the main topics of the study of the general circulation. Here we are interested in the temporally (i.e. monthly) averaged fields of wind, temperature, humidity, precipitation, and other meteorological variables and their long-term variations (also called *low-frequency variability*). For example, monsoon systems are a very important feature of the general circulation. For example, on the web-page http://users.ictp.it/~ kucharsk/speedy8_clim.html we find some features relevant to the general circulation.

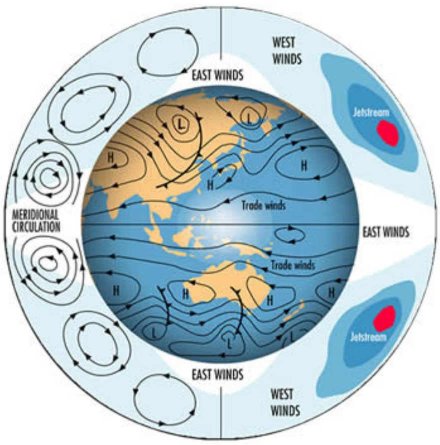

Figure 36: Schematic of some features of the general circulation.

6.1 Zonally averaged circulation

The aim of this section is to analyse the zonal mean circulation. The basis for the following analysis are the thermo-hydrodynamic equations in pressure coordinates Eqs. (33, 35, 36 and 37)

We apply in the following an averaging operator to these equations

$$\overline{A} \equiv \frac{1}{2\pi r cos\phi} \int_0^{2\pi} A \; r cos\phi d\lambda \quad . \tag{124}$$

All quantities are then expressed as the zonal mean plus a deviation from the zonal mean $A = \overline{A} + A'$.

For the total derivative of a quantity A

$$\frac{dA}{dt} = \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial p} \right) A + A \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} \right) \; , \tag{125}$$

where we added zero on the rhs according to the continuity equation. Therefore we may write the total derivative (in pressure coordinates!) as

$$\frac{dA}{dt} = \left(\frac{\partial A}{\partial t} + \frac{\partial Au}{\partial x} + \frac{\partial Av}{\partial y} + \frac{\partial A\omega}{\partial p} \right) \ , \tag{126}$$

Application of the zonal operator (124) yields

$$\overline{\frac{dA}{dt}} = \left(\frac{\partial \overline{A}}{\partial t} + \frac{\partial (\overline{A}\overline{v} + \overline{A'v'})}{\partial y} + \frac{\partial (\overline{A}\overline{\omega} + \overline{A'\omega'})}{\partial p} \right) \ , \tag{127}$$

because $\overline{\partial () \partial x} = 0$ and

$$\overline{ab} = \overline{(\overline{a} + a')(\overline{b} + b')} = \overline{\overline{a}\overline{b}} + \overline{\overline{a}b'} + \overline{a'\overline{b}} + \overline{a'b'} = \overline{a}\overline{b} + \overline{a'b'} \ ,$$

because the quantities $\overline{()}$ are independent of x and $\overline{a'} = \overline{b'} = 0$. Applying the zonal average to the continuity equation leads to

$$\frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{\omega}}{\partial p} = 0 \quad . \tag{128}$$

Note that with Eq. 128 we can define a streamfunction:

$$\Psi = \int_0^p \overline{v} dp \quad , \tag{129}$$

so that

$$\overline{v} = \frac{\partial \Psi}{\partial p} \quad ; \overline{\omega} = -\frac{\partial \Psi}{\partial y} \ . \tag{130}$$

To show 130 will be an Exercise! From Eq. (127) we can also derive

$$\overline{\frac{dA}{dt}} = \frac{\overline{d}}{dt}\overline{A} + \frac{\partial \overline{A'v'}}{\partial y} + \frac{\partial \overline{A'\omega'}}{\partial p} \ , \tag{131}$$

where

$$\frac{\overline{d}}{dt} = \frac{\partial}{\partial t} + \overline{v}\frac{\partial}{\partial y} + \overline{\omega}\frac{\partial}{\partial p} \tag{132}$$

is the rate of change following the mean motion. Averaging the zonal component of the momentum equation 33 and the thermodynamic equation 37 leads to

$$\frac{\partial \overline{u}}{\partial t} - f_0 \overline{v} = -\frac{\partial \overline{u'v'}}{\partial y} \tag{133}$$

$$\frac{\partial \overline{T}}{\partial t} - S_p \overline{\omega} = -\frac{\partial \overline{v'T'}}{\partial y} + \frac{\overline{Q}}{c_p} \tag{134}$$

Here several further approximations have been introduced which are all consistent with quasi-geostrophic scaling. A similar scaling shows that the meridional momentum equation is in quasi-geostrophic balance. For the zonal averaged meridional momentum equation, the first order geostrophic approximated balance is

$$f_0 \overline{u} = -\frac{\partial \overline{\Phi}}{\partial y} \quad . \tag{135}$$

Together with the zonal average of the hydrostatic equation 35 this leads to the thermal wind equation for zonal averaged motion

$$\frac{\partial \overline{u}}{\partial p} = \frac{R}{f_0 p} \frac{\partial \overline{T}}{\partial y} \quad . \tag{136}$$

Equations (133) and (134) tell us that in order to get a steady-state meridional, vertical circulation cell $(\overline{v}, \overline{\omega})$ we must have the balances

Coriolis force $f_0 \overline{v} \approx$ divergence of eddy momentum fluxes

Adiabatic cooling $\approx$ diabatic heating plus convergence of eddy heat fluxes

Also note that any $\overline{v}$ and $\overline{\omega}$ separately induces the other due to continuity 128.

Analysis of observations shows that outside the tropics these balances appear to be approximately true above the boundary layer. Close to the equator we have that the heating is mainly balanced by mean vertical motion, driving the *Hadley Cell*, whereas in the extratropics the meridional, vertical circulations are mainly driven by the convergence of eddy momentum and eddy heat fluxes. These cells are called *Ferrell Cells*. Discuss that the (angular) momentum fluxes should be toward the extratropics because of the absolute (angular) momentum loss af the atmosphere in the extratropics and gain in the tropics (Fig. 38, upper panel). Also discuss effect of tilt of waves, and the fact that Rossby waves radiate energy away from the jet (baroclinic zone), means at the same time that they carry momentum towards the jet (see Fig. 39) One can use the radiation condition considering the meridional energy propagation Eq. 48, suggesting $kl > 0$ (and $l > 0$) north and $kl < 0$ (and $l < 0$) south of jet, implying $k > 0$ in both cases)! Also discuss how the shear induced by the jet as well as $\beta(y)$ may modify the tilt of the phases of waves, with the latter one responsible for the dominance of poleward eddy momentum transport south of the jet.

Exercises

1. Show that with the streamfunction definition Eq. 129, the relationships 130 are fulfilled.

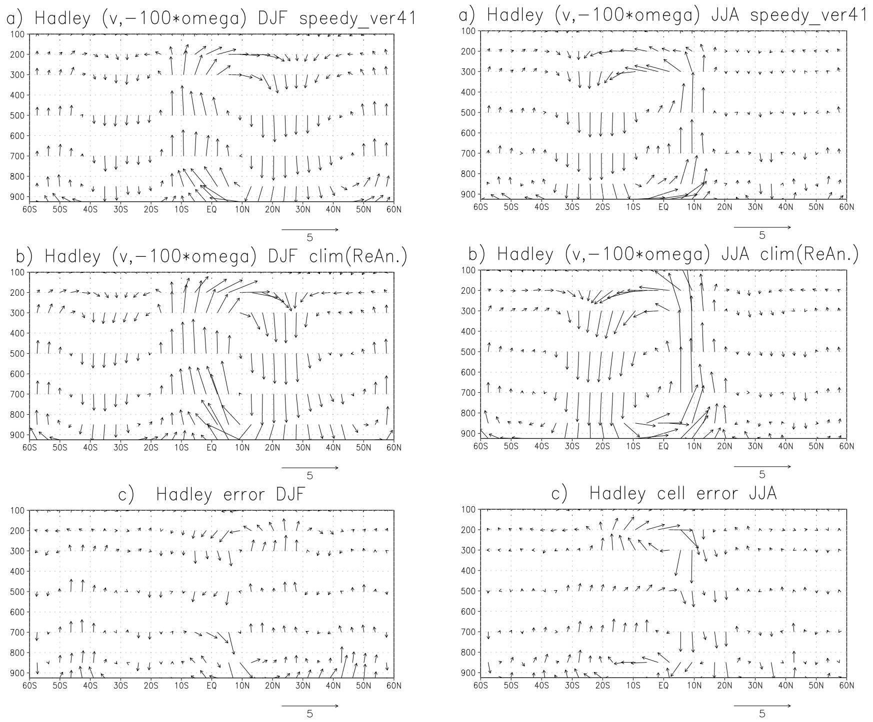

Figure 37: Illustration of the Hadley cell by a $(\overline{v}, -\omega)$ vector plot. left panel: Boreal winter, right panel: boreal summer.

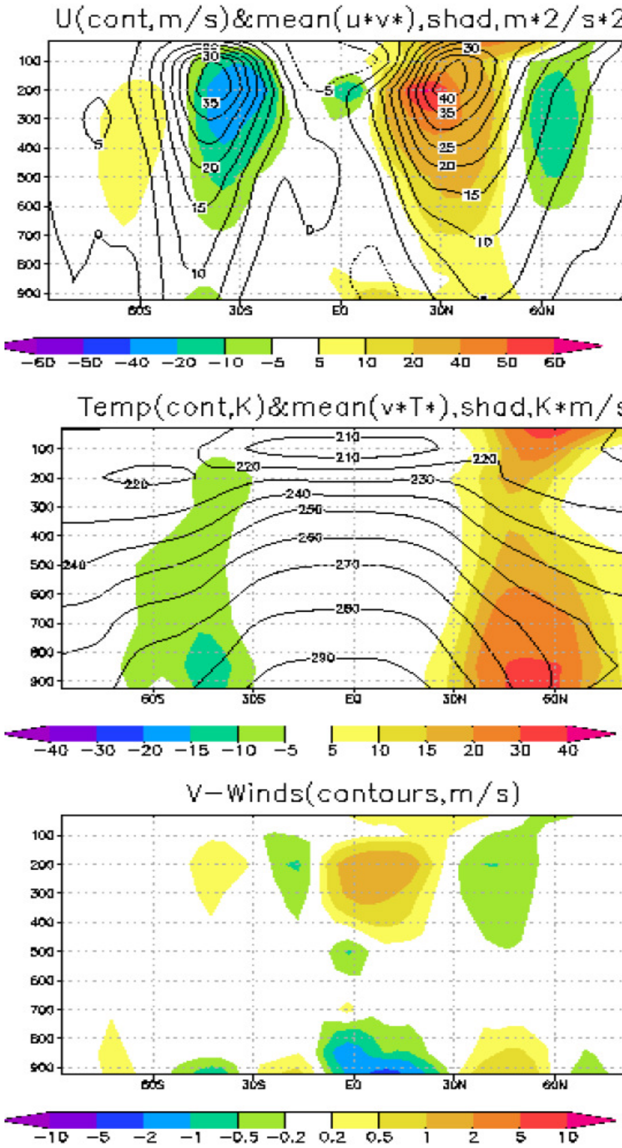

Figure 38: Upper panel: $\overline{u}$ (contours), and $\overline{u'v'}$, middle panel: $\overline{T}$ (contour) and $\overline{v'T'}$, lower panel $\overline{v}$.

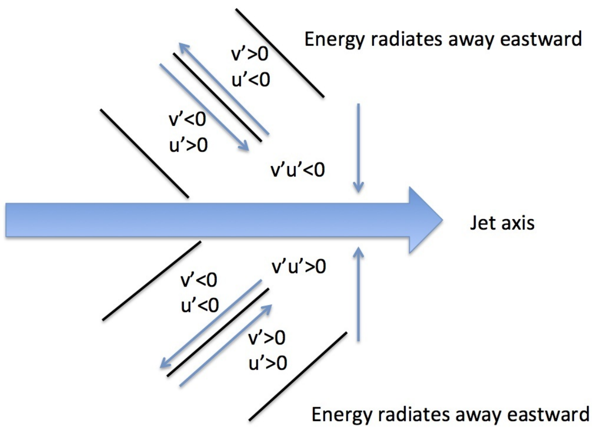

Figure 39: Sketch of why the eddy momentum flux is always towards the Jet.